ARAB REPUBLIC OF EGYPT **(١٣١١) ث . ع / أول / E**

Ministry of Education

General Secondary Education Certificate Examination, ٢٠١٨

{ First Session }

Pure Mathematics (Differentiation & Integration) **Time: ٢ Hours**

الرياضيات البحتة – التفاضل والتكامل 【 باللغة الإنجليزية 】 **(الأسئلة فى احدى عشر صفحة)**

تنبيه مهم : **يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة .**

Note: **Calculators are allowed.**

First: Choose the correct answer from those given:

(١) If $y = x^{٢} \sec \frac{١}{x}$, then $\frac{dy}{dx}$ = ...

a) $٢x \sec \frac{١}{x} + x^{٢} \sec \frac{١}{x} \tan \frac{١}{x}$

b) $٢x \sec \frac{١}{x} + \sec \frac{١}{x} \tan \frac{١}{x}$

c) $٢x \sec \frac{١}{x} - \sec \frac{١}{x} \tan \frac{١}{x}$

d) $٢x \sec \frac{١}{x} - x^{٢} \sec \frac{١}{x} \tan \frac{١}{x}$

(٢) If $x^{٢} + y^{٢} = ٢ \; x\, y$, the $\frac{dy}{dx}$ = ...

a) ٢ **b)** ١ **c)** zero **d)** -١

(٣) If $x = ٥ + ٣ \sin \mathbf{c}$, $y = ٢ \cos \mathbf{c}$, where $\mathbf{c} \in [\, ٠, \frac{\pi}{٢} \,]$, then the curve has a horizontal tangent at the point .....

a) (٥ , ٢) **b)** (٥ , ٣) **c)** (٨ , ٢) **d)** (٢ , ٢)

[بقية الأسئلة فى الصفحة الثانية]

(٤) $\lim_{x\to ٠}(١+x)^{\frac{١}{٣x}}$ = ...

a) $e^{٣}$ **b)** $\frac{e}{٣}$ **c)** $e^{\frac{١}{٣}}$ **d)** $\frac{١}{٣}$

(٥) If $f(x) =$ ln sin x - ln cos x , then f " $(\frac{\pi}{٤})$ = ...

a) ٢ **b)** -٤ **c)** -١ **d)** zero

(٦) $\int ٤\, x\, e^{x^{٢}}\, dx$ = ...

a) $\frac{١}{٢}\, e^{x^{٢}} + c$

b) $e^{x^{٢}} + c$

c) $٤\, e^{x^{٢}} + c$

d) $٢\, e^{x^{٢}} + c$

[بقية الأسئلة فى الصفحة الثالثة]

(٧) If $f(x) = e^{٨x - x^{٢}}$, then the function is increasing in the interval .....

a)] – ١ , ٠ [**b)**] ٠ , ١[**c)**] ٤ , ١[**d)**] – ١ , ٤ [

(٨) If $f(x) = x^{٣} + kx^{٢} + ٤$ and the curve of the function has an inflection point at $x = ١$, then k =.....

a) -٦ **b)** -٣ **c)** ٣ **d)** ٦

(٩) $\int_{-١}^{١} \frac{x^{٣}}{x^{٤}+\cos x}\, dx$ = ...

a) -١ **b)** $zero$ **c)** ١ **d)** ٤

(١٠) $\int_{\square}^{\square} (٣x + ٢)\ sin\ x\ dx$ = ...

a) $(٣x + ٢)\cos x + ٣\sin x + c$

b) $-(٣x + ٢)\cos x + ٣\sin x + c$

c) $(٣x + ١)\cos x + ٢\sin x + c$

d) $-(x + ١)\cos x - ٢\sin x + c$

[بقية الأسئلة فى الصفحة الرابعة]

Second: Answer the following questions:

(١) If $xy = \sin^{٢} x$, prove that :

$$x \frac{d^{٢}y}{dx^{٢}} + ٢ \left(\frac{dy}{dx}\right) + ٤xy = ٠ \quad \text{.....}$$

Complete: $\because xy = \sin^{٢} x$

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(٢) If the slope of the tangent to the curve of the function f at any point (x, y) on it equals $٢\, e^{-\frac{١}{٢}x}$ and $f(٠)=١$, find $f(٣)$

Complete: $f(x) = ٢\, e^{-\frac{١}{٢}x}$

[بقية الأسئلة فى الصفحة السادسة]

(٣) Study the changes of the function f where $f(x) = x^{٣} - ٦x^{٢} + ٩x - ١$ indicating the local maximum, the local minimum points and the points of inflection if they exist.

Complete:

$\because f(x) = x^{٣} - ٦x^{٢} + ٩x - ١$

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[بقية الأسئلة فى الصفحة السابعة]

(٤) Find $\int x\,(x-٢)^{٤}\,dx$

Complete:

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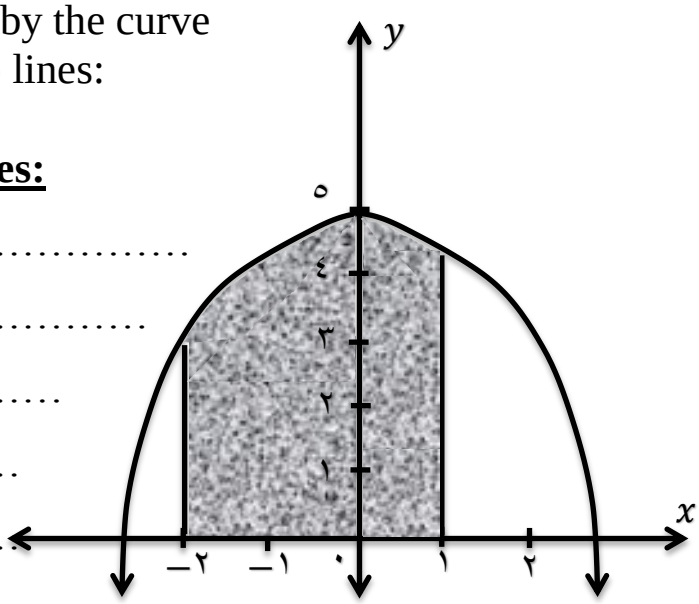
[بقية الأسئلة فى الصفحة الثامنة]

(٥) Find the area of the region bounded by the curve $y = ٥ - x^{٢}$ and the x-axis and the two lines:
$x = -٢$, $x = ١$

Complete using the opposite figures:



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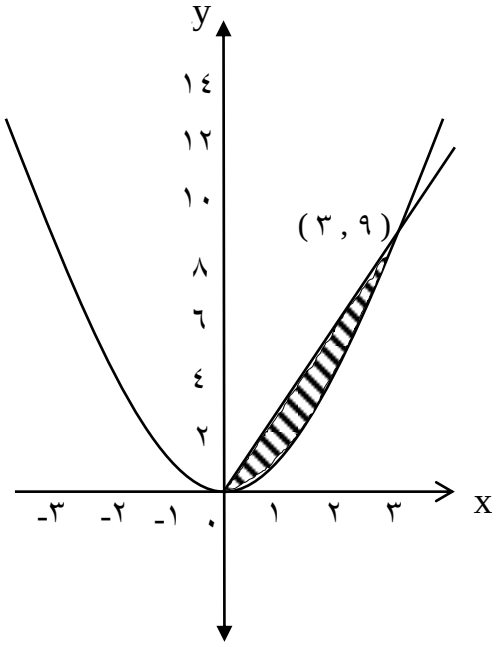
[بقية الأسئلة فى الصفحة التاسعة]

(٦) Find the volume of the solid generated from revolving the area bounded by the curve $y = x^{٢}$ and the line $y = ٣x$ a whole revolution about the *x –axis* .

Complete using the opposite figure:



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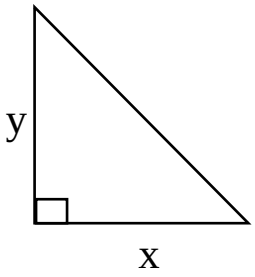
[بقية الأسئلة فى الصفحة العاشرة]

(٧) At a certain moment the side lengths of the right angle of a right-angled triangle were ٨ cm and ٦ cm. If the length of the first side was decreasing at a rate of ١ cm/min and the length of the second side was increasing at a rate of ٢ cm/min, find the rate of change of the area after ٢ min. When does the rate of change of the area vanish?



Complete:

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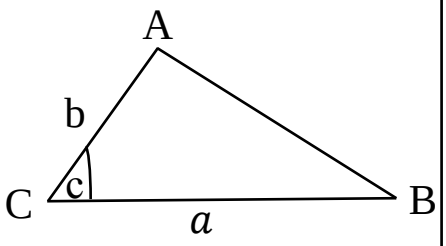
[بقية الأسئلة فى الصفحة الحادية عشر]

(٨) ABC is a tringle where a, b are constant. Find the measure of the included angle between them which makes the area maximum.



Complete:

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[أنتهت الأسئلة]